Economic Inequality, Political Inequality, and Redistribution

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Abstract

Citizens are not politically equal in economically unequal societies. When political influence of individuals increases in their income and political decisions are made by coalitions with greater political influence, the decisive agent has income higher than the median. Policy consequences are far reaching. In particular, the extent of redistribution of income through taxes and transfers is not only always lower than the rate desired by the citizen with the median income but when political influence is sufficiently sensitive to income, the rate of redistribution falls when income inequality increases. The mechanism that generates this pattern is competition among interest groups for political influence. The end result is that representative institutions do not mitigate economic inequality, as they would in politically egalitarian systems. Yet some extent of political inequality is inexorable in economically unequal societies.

> "The state abolishes, in its own way, distinctions of birth, social rank, education, occupation, when it declares that birth, social rank, education, occupation, are non-political distinctions, when it proclaims, without regard to these distinctions, that every member of the nation is an equal participant in national sovereignty.... Nevertheless the state allows private property, education, occupation to act in their way – i.e., as private property, as education, as occupation, and to exert the influence of their special nature." (Marx 1844)

1 Introduction

Citizens are not politically equal in economically unequal societies. While political equality is an attractive normative principle, *pace* Downs (1957: 32-33), the assumption that "the preferences of no one citizen are weighted more heavily than the preferences of any other citizen" (Dahl and Lindblom 1953: 41) cannot serve as a point of departure for positive analysis. Yet, ever since Black (1948), political economists have relied almost exclusively on a model that assumes political equality of all citizens and implies that the decisive actor is the one with the median preference, a model extended in the context of income by Romer (1975), Roberts (1977), and Meltzer and Richards (1981) to identify this actor as the one with median income.[1] This entire construction is a house of cards: the assumption of political equality flies in the face of everyday experience while empirical tests of such models fail miserably.

Explanations of why the median voter theorem fails come in droves (For overviews, see Putterman (1996), Harms and Zink (2003), Lind (2005), Ansell and Samuels (2010)). Roemer (1998) shows that the rate of redistribution that emerges from electoral competition is lower than the one desired by the median voter when the competition entails a dimension other than economic. Huber and Staning (2003), Goskens, Golder, and Siegel (2005), as well as Stemueller (2013) single out religion as the second dimension, while Dion (2010) invokes not specific religions but religious or ethnic fragmentation. Piketty (1995), Fong (2001), and Alesina and Angeletos (2005) argue that people vote according to their norms of fairness, applying beliefs about whether incomes are generated by effort or luck. Bénabou and Ok (2001) believe that people vote according to their expectations of upward social mobility. Corneo

[1] Bertola (1993) extended this model to the growth framework. Acemoglu and Robinson (2000) used it explain franchise extensions. Rosendorf (2001) applied it to regime transitions. These are just a few examples: applications of this model must run in hundreds, if not thousands.

and Gruner (2000) maintain that the median voter is concerned about her social status and wants to preserve their distinction from the poor. Przeworski, Rivero, and Xi (2015) assume that feasible redistributions are constrained by the threat of violent conflicts. Finally, Bartels (2005) provides evidence that voters who hold egalitarian views often do not understand which policies would implement them. My claim is that all these are second-order reasons: the basic assumption that is wrong with the median voter model is that citizens are politically equal. Hence, I join Bassett, Burkett, and Putterman (1999), Bénabou (2000), Bernagen and Bräuninger (2005), and Kelly and Enns (2010) in examining the effects of political inequality.

I show that when political influence of individuals increases in their incomes and political decisions are made by coalitions with greater political influence, the decisive agent has income higher than the median. Policy consequences are far reaching. In particular, the extent of redistribution of income through taxes and transfers ("the fisc") is not only always lower than the rate desired by the citizen with the median income but, when political influence is sufficiently sensitive to income, the rate of redistribution falls, rather than increase, when income inequality increases. The mechanism that generates this pattern is competition among interest groups for political influence. The end result is that representative institutions do not mitigate economic inequality, as they would in politically egalitarian systems. Some extent of political inequality, however, is inexorable in economically unequal societies.

In what follows, I work backwards. First, I analyze the functioning of representative institutions just assuming that people with higher incomes exert greater political influence. Only then, I study some mechanisms by which economic inequality results in political inequality: lower rates of political participation among people with lower incomes and competition for political influence. The concluding section is a call to rethink other policies through the prism of political inequality.

2 Political Inequality

This section proves and illustrates the claim that when agents with higher incomes have more political influence and collective decisions are made by the coalition majoritarian in political influence, the decisive agent has income higher than the median and her location in income distribution increases as the effect of income on political influence becomes larger.

In what follows, y^i stands for income of $i \in [0, 1]$ and $F(y)$ is some continuos, unimodal, right-skewed distribution of income.

Proposition 1 *Assume that income is distributed according to* $p = F(y)$*, where* p *stands for the location of an individual in the distribution. When all agents have equal political influence, the decisive agent is the agent with the median income,* $p = 0.5$*. Given any "political influence function"* $w(y) = y^{\eta}$*, with constant elasticity* η*, the decisive agent* $i = D$ *has* $p^D > 0.5$ *for any* $\eta > 0$*. Moreover, the higher the* η*, the higher the* p^D *of the decisive agent.*

Proof. Consider any distribution of income $F_Y(y)$ and a monotonically increasing function of income, $w(y)$, where w stands for "political weight." Then the random variable $w(y) \sim F_W$ with $F_W := F_Y \circ w^{-1}$,

$$\begin{aligned} F_W(k) &= \mathbf{p}(\{w : w \leqslant k\}) = \mathbf{p}(\{y : w(y) \leqslant k\}) = \mathbf{p}(\{y : y \leqslant w^{-1}(k)\}) \\ &= F_Y(w^{-1}(k)) = F_Y \circ w^{-1}(k). \end{aligned}$$

Assume that collective decisions are made by a coalition majoritarian in political influence and consider a value of $w = w^D$, where D stands for "decisive," such that $\int_0^{F_W(w^D)} \inf(F_W)^{-1}(t)dt = \int_{F_W(w^D)}^1 \inf(F_W)^{-1}(t)dt$. By definition of Lorenz curve, $L(F_W(w^D)) = 0.5$. Let $w = y^{\eta}$. The distribution of political influence is then $F_W(w) = F_Y(y^{\eta})$ and $F(\eta = 0) \geq_2 F(\eta > 0)$, where $\geq_2$ stands for second-order stochastic dominance. Use now the result of Thistle (1989, Proposition 4) that if distribution F_1 second-order stochastically dominates F_2, then F_1 Lorenz dominates F_2, i.e. $L_1(p) \geqslant L_2(p)$ $\forall p \in [0, 1]$. Hence, $L(\eta = 0) \leq L(\eta > 0)$, so that $p^D(\eta = 0) \leq p^D(\eta > 0)$. It is obvious that when $w(y) = 1, \forall y$, $L(p^D) = 0.5$ when $p(y^D) = 0.5$. Because $p^D(\eta = 0) = 0.5$, $p^D(\eta > 0) > 0.5$. ■

The intuition is embarrassingly simple. The winning coalition is the one for which the sum of political weights is greater. Hence, the decisive agent is the one whose inclusion determines which coalition – of agents with incomes lower or higher than she – has larger influence. Take five agents with weights $\mathbf{w} = \{1, 2, 3, 4, 5\}$. The possible coalitions majoritarian in political influence are then $\{1, 2, 3, 4\}, \{4.5\}$ and because the agent with $w = 4$ must be included in any majority coalition, this agent is decisive.[2] Now, hold the distribution of income constant and compare distributions of political influence differing in the elasticity of the influence function. The Lorenz curve for higher elasticity lies below with lower elasticity. Hence, the p^D for which $L(p) = 0.5$ must be higher for the more unequal distribution of political influence. Figure 1 shows the

[2] Because w increases monotonically in y and because preferences over r depend only on income, the order restriction of Rothstein (1991) holds, which implies in turn that any coalition can contain only individuals with adjacent incomes.

effect of political inequality holding income distribution constant.[3]

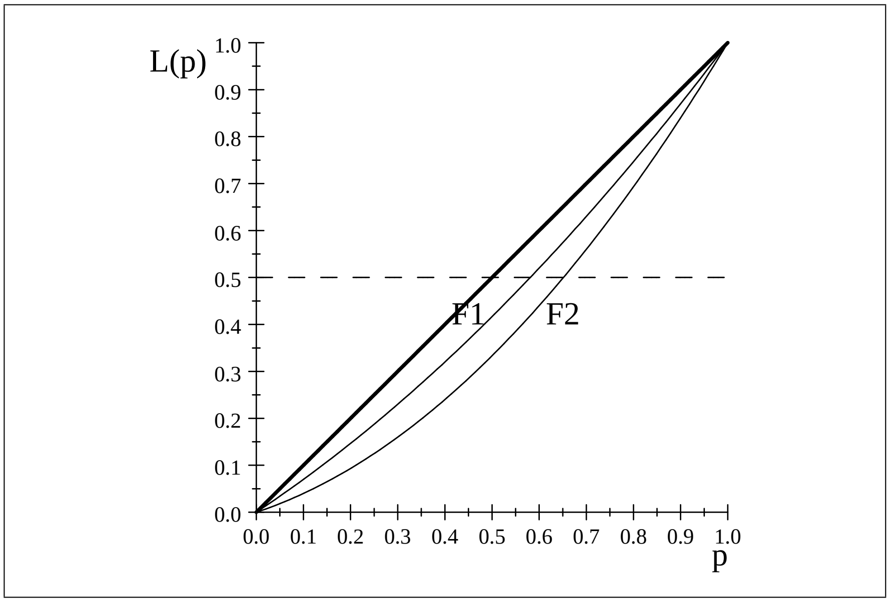


Figure 1: Percentile locations of the decisive agent as a function of inequality of political influence, given income distribution.

Thus, for any distribution of income, political inequality places the decisive political influence in the hands of an agent with income higher than the median.

3 Political Inequality and Redistribution

The main implication of the median voter model is that the rate of redistribution through the fisc increases in income inequality. Suppose each agent $i \in [0,1]$, with pre-fisc income y^i, solves the problem

$$\arg\max_r \{(1-r)y^i + r(1-\lambda r)\overline{y}\}, \tag{1}$$

where r is the rate of redistribution,[4] λ represents static deadweight loss, due to labor supply, administrative costs, waste, corruption, etc., and $\overline{y}$ is the average income. The solution to this problem is

$$r^i = \begin{array}{ccc} 0 & if & y^i \geq \overline{y} \\ \frac{\overline{y}-y^i}{2\lambda\overline{y}} & if & 0 < 1 - y^i/\overline{y} < 2\lambda \\ 1 & if & 1 - y^i/\overline{y} > 2\lambda \end{array} \tag{2}$$

[3] Obviously, the same is true if political elasticity is constant but income inequality is higher.

[4] I deliberately write the rate of redistribution as r, rather than τ, because tax revenues are used for many purposes other than redistribution.

Now, under the egalitarian mechanism the decisive agent is the agent with the median income, to be denoted as $i = M$. Hence, in the interior, $r^M = \frac{1-y^M/\overline{y}}{2\lambda}$ and the rate of redistribution moves in the same direction as income inequality, indicated by δ, whether $1 - y^M/\overline{y}$ or the Gini coefficient.

Consider now inegalitarian mechanisms, for which political influence is given by $w = y^\eta$, $\eta > 0$, and collective decisions are made by the coalition majoritarian in influence, The decision about redistribution is now made by the agent for whom $L(F_W(w)) = 0.5$, with income y^D. This agent chooses (in the interior)

$$r^D(\eta, \delta) = \frac{1}{2\lambda}(1 - \frac{y^D(\eta, \delta)}{\bar{y}}). \tag{3}$$

The following result can be proved for the two distributions commonly used to characterize distributions of income: lognormal and Pareto.

Proposition 2 *If the distribution of income is lognormal, there exists a value $\eta^* = 2^{-1/2} \approx 0.71$ such that if $\eta < \eta^*$, the rate of redistribution increases and if $\eta > \eta^*$ it decreases in income inequality. If income distribution is Pareto, the rate of redistribution increases monotonically in income inequality if $\eta < \eta^* \approx 0.78$ and it has an internal maximum if $\eta > \eta^*$. Moreover, the maximum occurs at a lower level of inequality, so that the rate of redistribution decreases in a broader range of income inequality when η is larger.*

Proof. In the Appendix. ■

The intuition behind this result is that as income inequality increases, the ratio of median to mean income decreases but the location of the decisive agent in income distribution increases. When η is sufficiently high, the second effect dominates the first, so that the ratio $y^D/\bar{y}$ increases and r^D decreases. These two effects are portrayed in Figure 2 for a Pareto distribution and $\eta = 0.8$.[5]

[5]The range of Gini coefficients of gross income given by SWIID (2011) is from 0.17 in Bulgaria in 1968 to 0.79 in the Maldives in 1998. I present most results in this range.

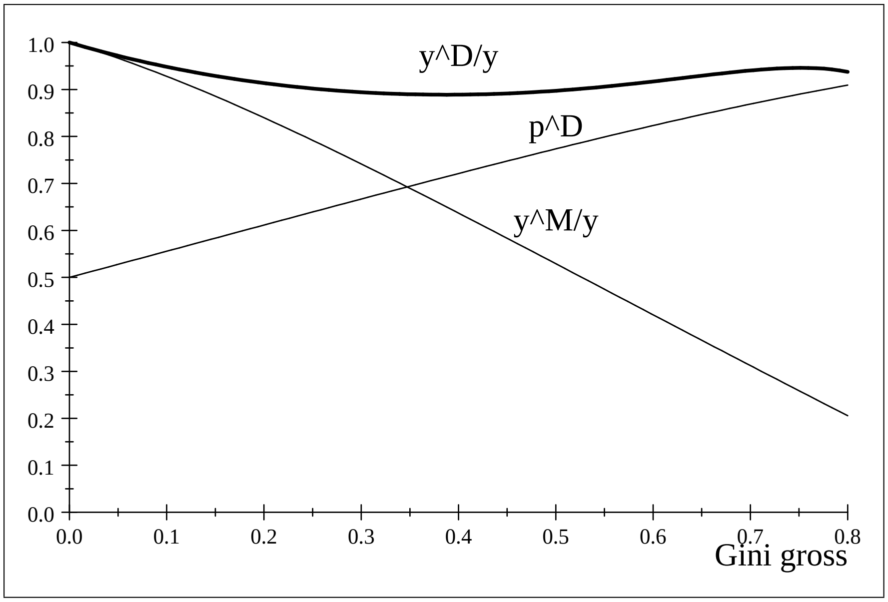


Figure 2: Effects of income inequality on the ratio of the median to mean income ($y^M/\bar{y}$), the location of the decisive agent in income distribution (p^D), and their combined effect on the ratio of the income of the decisive agent to mean income ($y^D/\bar{y}$).

Figure 3 shows the rates of redistribution chosen by the decisive agent given that the distribution of income is Pareto, for different elasticities of the influence function. Reading it vertically shows that, for any income distribution, the rate of redistribution is lower when the elasticity is larger. The thick line is the benchmark, namely, complete political equality, while the lines below are for $\eta = \{0.5, 0.8, 1\}$. Strikingly, the function changes shape when political inequality becomes sufficiently large, $\eta \approx 0.78$: not only is the rate of redistribution lower but in some range of inequality it decreases as inequality of market incomes becomes larger.

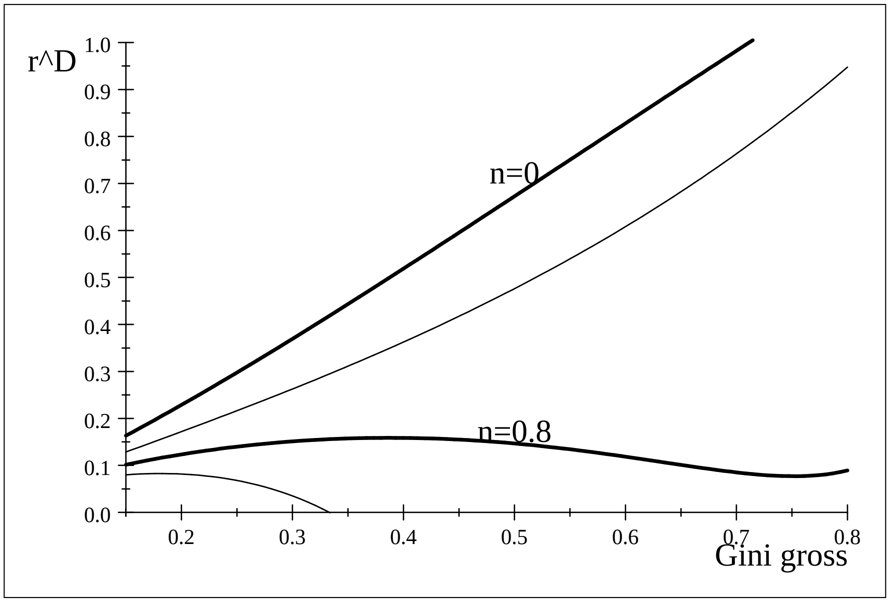


Figure 3: The rate of redistributon of the decisive agent as a function of income inequality, for different elasticities of the political influence function, given a Pareto distribution. ($\lambda = 0.35$)

Now, almost all[6] the criticisms of the median voter model cited in the Introduction explain why observed the rates of redistribution are lower than those predicted by this model. Yet when political inequality is sufficiently high relative to economic inequality, not only are the rates lower but they fall as income inequality increases. This fact has profound consequences, for it implies that, contrary to the ingrained beliefs about democracy, under such conditions representative institutions do not mitigate the inequality generated by markets.

If the median voter were decisive, the distribution of post-fisc incomes would remain relatively stable regardless of the distribution of market incomes: the egalitarian political mechanism would mitigate economic inequality by increasing taxes and transfers. Note that the rate of redistribution implicit in expression (1) can be written as $r = (G^M - G^N)/G^M$, where G^M and G^N are, respectively, Gini coefficients of market ("gross," "pre-fisc") and net ("disposable," "post-fisc") incomes. Hence, $G^N = (1-r)G^M$. The effect of market inequality on the distribution of net incomes is portrayed in Figure 4, for $\eta = \{0.5, 0.8\}$. When the elasticity of influence with regard to income is sufficiently high, the

[6] One exception is Moene and Wallerstein (2001), who distinguish income and insurance motives in voting decisions and show that when transfers are directed to the unemployed, social expenditures decline in inequality. Another exception is Benabou (2000).

Gini of net incomes rises monotonically in the Gini of market incomes, so that redistribution does not play a corrective role. The line for $\eta = 0.8$ is singled out for two reasons: (1) according to the Proposition 2 (see the Appendix), the function changes shape just below this value, (2) as shown below, this value of η provides an almost perfect fit to the data.

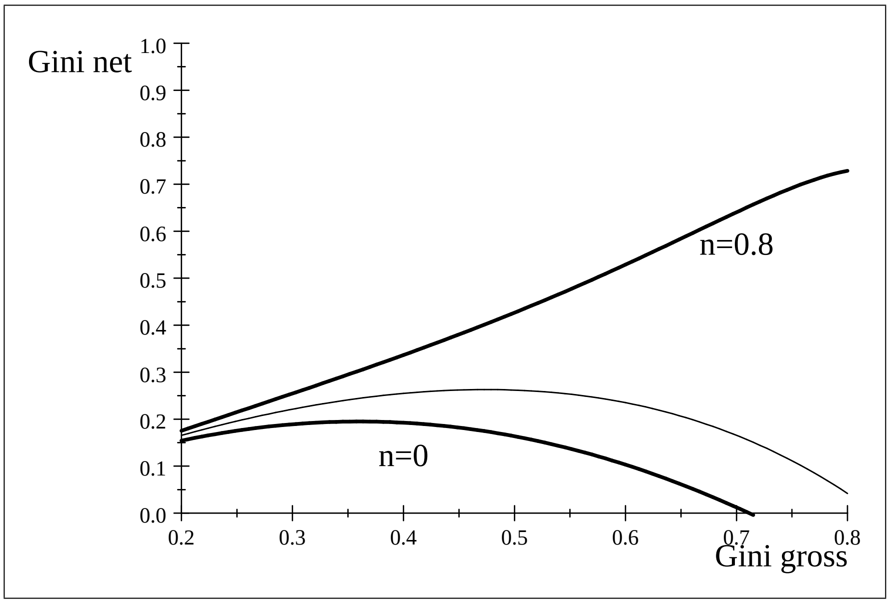


Figure 4: Gini of net incomes as a function of Gini of market income, given different elasticities of the political influence function ($\eta = 0, 0.5, 0.8; \lambda = 0.35$).

Note: Given Pareto distribution, $G = (2\alpha - 1)^{-1}$, so that $\alpha = (1 + G)/2G$. When decisions are made by the voter with median income, $r^M = \frac{1-y^M/\overline{y}}{2\lambda}$, where $y^M = 2^{1/\alpha} = 2^{2G/(1+G)}$ and $\overline{y}^{-1} = \frac{\alpha-1}{\alpha} = \frac{1-G}{1+G}$. Hence, $r^M = \frac{1-\frac{1-G}{1+G}2^{2G/(1+G)}}{2\lambda}$. In turn, $r^D = \frac{1-\frac{\alpha-1}{\alpha}0.5^{1/(\eta-\alpha)}}{2\lambda} = \frac{1-\frac{1-G}{1+G}0.5^{1/(\eta-\frac{1+G}{2G})}}{2\lambda}$. The figure plots $G^N = (1 - r^D)G^M$ for $\lambda = 0.35$.

The fit of the model calibrated for $\eta = 0.8$ to the inequality data from the SWIID (2014) data base is shown in Figure 5. The straight line is the prediction of the model, the gray area is the 95 percent confidence interval of fractional polynomial function fitted to the data.

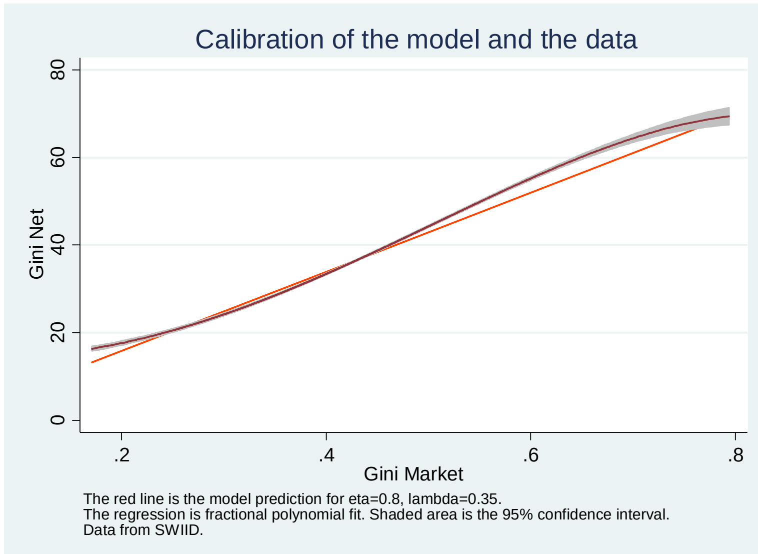


Figure 5

Hence, it appears that the observed world exhibits evidence for extensive political inequality.

4 Economic Inequality and Political Inequality

Formal political equality – defined Beitz (1989: 4) as institutions that provide citizens with equal procedural opportunities to influence political decisions – is not sufficient to generate equality of actual influence over the outcomes because effective political equality depends on the distribution of the enabling resources. Wealth or income may affect political influence through several channels, with stronger or weaker effects on political inequality.[7] I focus on two mechanisms by which differences in income may affect policy outcomes: (1) Even when they have equal rights, some people may not enjoy the material conditions necessary to participate in politics, (2) The competition among interest groups for political influence may lead policy makers to favor larger contributors. I show that when the poorest people are unable to vote, the rate of redistribution is always lower than it would be if everyone participated, but still increases monotonically in income inequality. Competition for

[7] Obviously, income (or wealth) is not the only potential source of political inequality: the military may be politically influential because they have guns; co-ethnics of a political leader may have privileged access to the government (De Luca et al. 2015); occupations that produce knowledge may have more authority over some policy realms, etc. But the relevant question here is only whether income differences must be reflected in the inequality of influence over decisions made by governments.

political influence among agents with different incomes, however, does generate a pattern specified in Proposition 2.

4.1 The Effect of Social Conditions

Political inequality may emerge in economically unequal societies without anyone doing anything to enhance their influence or reduce the influence of others, just because some people may not enjoy the material conditions necessary to exercise their political rights. Rights to act are hollow in the absence of the enabling conditions (J.S. Mill 1857, Holmes and Sunstein 1999, Sen 1998).

The simplest way of thinking about political inequality when some people do not have the conditions to exercise their formal rights is that they cannot participate in political activities unless they enjoy some minimum income, say $\underline{y}$. Then people with $y < \underline{y}$ have the political weight of 0, while everyone above this threshold has a weight of 1. While it extends to other forms of political activity, this effect of social conditions is most apparent in the fact that in most, albeit not all,[8] countries poorer people tend to vote at lower rates.

Remark 1 *Given a Pareto distribution of income, when agents with lowest incomes do not vote and their proportion is* $1-t$*, the vote maximizing rate of redistribution is* $r^o = \frac{1}{2\lambda}(1-(1-t)^{-1/\alpha}2^{1/\alpha}\frac{\alpha-1}{\alpha})$.

Proof. Given t, the income of the poorest voter who participates is given by $(1-\underline{y}^{-\alpha}) = t$, so that this income is $\underline{y} = (1-t)^{-1/\alpha}$. Median income among participants is then $(1-t)^{-1/\alpha}2^{1/\alpha}$. If the rate of redistribution is decided by the voter with the median income among the participants, the vote maximizing rate is r^o. Given $G = (2\alpha-1)^{-1}$, expressed in Ginis, $r^o = \frac{1}{2\lambda}(1-t^{-\frac{2G}{1+G}}2^{\frac{2G}{1+G}}\frac{1-G}{1+G})$. ■

Now I rely on an empirical observation based on the SWIID (2014) and PIPE (2014) data bases: linear regression of the rates of electoral turnout on Gini coefficients yields a coefficient of -0.85, with the 95% confidence interval $[-0.90, -0.81]$. Substituting this function into r^o (and using as always $\lambda = 0.35$) generates the pattern portrayed in Figure 6, where the thick line shows the rate of redistribution of the median among all agents

[8] Kasara and Suryanarayan (2015) show that the rich vote at rates lower than the poor when the salience of redistributive issues in politics or the state's extractive capacity is low. Otherwise, the rich vote at higher rates.

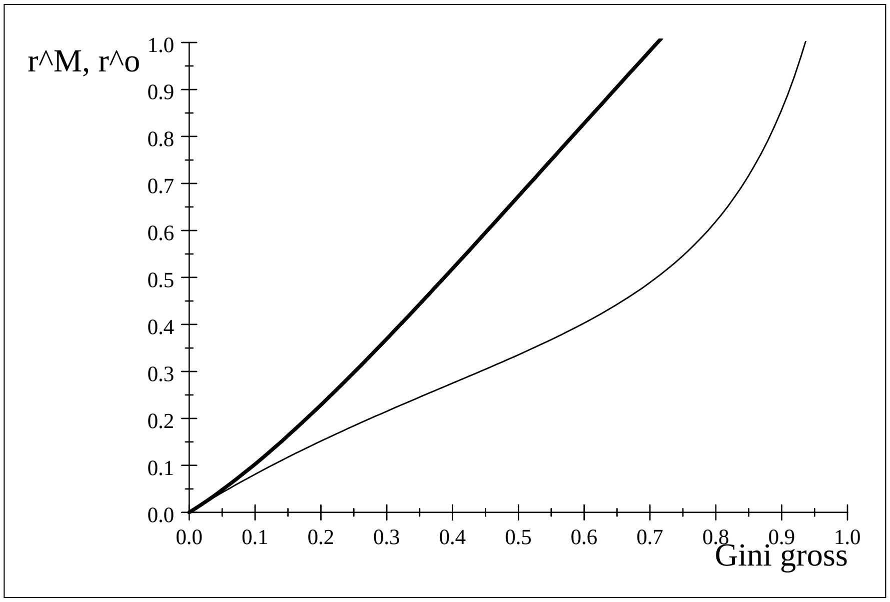


Figure 6: Vote maximizing rates of redistribution given Gini when 1-0.85*Gini participate.

To assess the effect of differential turnout rates on the rates of redistribution, we can use a survey conducted in 2012 in 24 European countries[9], showing that people in the lowest income decile voted at the rate of 69 percent, while those in the top decile at the rate of 85 percent, as well as data from the United States[10], where the rates of participation were 49 percent for people with incomes under $10,000 and 81 percent for those with incomes above $150,000 in 2008, and respectively 47 and 80 percent in 2012. Taking the political weight of the lowest income group as 1, calculating the weights of higher income groups as multiples of the turnout of the lowest group, and assuming that income distribution has a Gini coefficient of 0.4 shows that the rate of redistribution if everyone participated would have been $r^M = 0.52$, while the rate with unequal participation would be $r^o = 0.49$ in Europe and $r^o = 0.42$ in the U.S. Obviously, this is just a rough exercise but, as Figure 7 demonstrates, regression analysis shows that electoral participation, the ratio of voters to the population, has a powerful effect on the actual rates of redistribution.

[9] www.eldiario.es/piedrasdepapel/promesa-igualitaria-democracia_6_309429090.html. "La promesa igualitaria de la democracia." Ignacio Jurado. 02/10/2014

[10] www.demos.org/blog/10/30/14/how-reduce-voting-gap

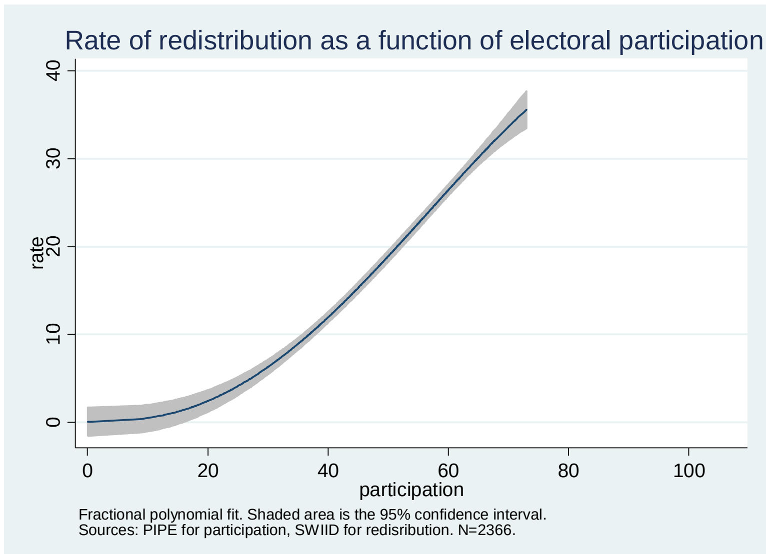


Figure 7: Electoral participation (the ratio of voters to population) and redistribution.

4.2 Competition for Political Influence

That groups compete for political influence, using various resources at their disposal, is the quintessence of modern political economy, a belief shared across the political spectrum (Stigler 1975, Habermas 1975). The only question is whether when the resources that individuals or groups command are unequal, the distribution of political influence resulting from competition for political influence must also be unequal; more narrowly here, whether and to what extent political influence that results from this competition is associated with income (or wealth).

Here is the structure of the argument that generates a positive answer to this question:

(1) Government policies affect the welfare of particular groups and individuals.

(2) Because government policies affect their welfare, groups and individuals want to influence these policies in their favor. To gain political influence, they are willing to incur costs that equalize at the margin the benefits from the resulting policies and the costs of buying influence (Becker 1983).

(3) Incumbent governments want to remain in office while opposition parties want to occupy it. While politicians and bureaucrats may have other motivations – they may seek private rents (Tullock 1967, Krueger 1974) or to maximize budgets (Niskanen 1971) – these assumptions are not necessary for what follows. The inescapable fact is that politics costs

money. Parties need money to exist, to organize election campaigns, to survey public opinion, to bring their supporters to the polls, to persuade those undecided to vote for them. They need to cover costs of meeting rooms, transportation, printing materials, access to television. Hence, even if all they want is to win elections, politicians may be willing to sell political influence, at least in the form of "access" but also directly in the form of policies.

(4) In the end, in an equilibrium, special interests exchange political contributions for political influence and government policies reflect the distribution of political influence.

Thus far, however, nothing guarantees that political influence would increase in income (or wealth) of the influencers. The Chicago School of Regulation, which saw this competition as one between consumers and producers (Posner 1974, Peltzman 1976) or between different groups of producers (Stigler 1971), concluded that the results are always detrimental to consumers but this conclusion was based on the argument that producers are willing to spend more because their benefits are concentrated while consumers' costs are diffuse. This line of analysis was generalized in the seminal model of competition for political influence by Becker (1983), who noted that while larger groups are more influential purely because of their size, they face more difficult collective action problems, and concluded that smaller groups compete more effectively. The bewildering aspect of Becker's model and the entire literature that followed (Austen-Smith 1997), however, is that income and wealth differences are assumed away.[11] Models of competition for political influence derive results with regard to the size of competing groups, their ability to control collective actions problems, the information that they control, but not income or wealth.

Consider, then, what happens when the competing groups differ in economic endowments and the policies that they seek to influence concern redistribution of income.

Order incomes in increasing magnitude and assume that income recipients organize themselves in groups, indexed by j, consisting of members with contiguous incomes. Given any r^o, the agents who would want the rate of redistribution to be higher are those with incomes $y^i \leq (1-\lambda r^o)\overline{y} \equiv y^o$: they are the "poor" here.[12] In turn, the "rich" agents are all those for whom $y^i > y^o$.

[11] "I have assumed that influence functions depend only on the characteristics of and the pressures exerted by political groups, and not on ... the distribution of income ...." (Becker 1983: 394).

[12] Remember that tranfers from or to are $T^i = r[(1-\lambda r)\overline{y} - y^i] \gtreqless 0 \iff (1-\lambda r)\overline{y} \lesseqgtr y^i$.

I analyze a game between all the poor against all the rich and allow that some of the rich may not oppose redistribution. To characterize these groups, I assume that their strategy is dictated by the agents who have mean incomes within them, ignoring the collective action issues that were the focus of Becker (1983) and others. The mean income of the poor is $\int_0^{y^o} yf(y)dy/F(y) \equiv y^P$ and of the rich is $\int_{y^o}^{\infty} yf(y)dy/[1-F(y)] \equiv y^R$. These values depend on the distribution of income, which for the Pareto distribution, which is used below, yields $y^P = (1-(y^o)^{-\alpha})^{-1}\int_1^{y^o} \frac{\alpha}{y^\alpha}dy$ and $y^R((y^o)^{-\alpha})^{-1}\int_{y^o}^{\infty} \frac{\alpha}{y^\alpha}dy$. Given the Pareto distribution and using as the benchmark the situation in which everyone votes and $r^o = r^M$, when the Gini coefficient is 0.50, the poor constitute 71 percent of the population and they have 34 percent of total income. (The corresponding numbers for $G = 0.4$ are 62 percent of the population and 38 percent of the income.)

These groups compete for political influence. Specifically, they are willing to contribute $x^j(r)$ when the government sets the redistribution rate at r. The rich are willing to pay more for lower r, the poor for higher r, so that it must be true that $\partial x^R/\partial r < 0$ and $\partial x^P/\partial r > 0$.

The government wants to be re-elected but also to receive contributions. I leave aside the question whether the government is venal, that is, just pockets the money at the cost of reducing its probability of re-election or uses the contributions of buy votes of "influenceable voters," as in Peltzman (1976), Becker (1983), and Grossman and Helpman (2001). The government's utility function has the general form of $G = G(r, \mathbf{x}(r))$, where $\mathbf{x}$ is a vector of contributions.

Under the assumptions of Grossman and Helpman (2001, Chapter 8), we can portray this game in the (r, x) space. The curve $G(\neg j)G(\neg j)$ shows the government indifference curve when group $j \in \{R, P\}$ – the rich R, the poor P, or both – did not make a contribution. The vote maximizing rate of redistribution based on policy alone is r^o. Because the government must be at least as well off under any contribution as it is when it just minimizes $|r - r^o|$ without contributions, this indifference curve must then pass through the point $(r^o, 0)$. The curves $U(j)U(j)$ show the indifference curves of these two groups: R is willing to increase its contributions in exchange for a lower r and P is willing to do the same in exchange for a higher r. The question to be answered is whether competition for influence between a poor and a rich group causes the government to increase or to decrease r.

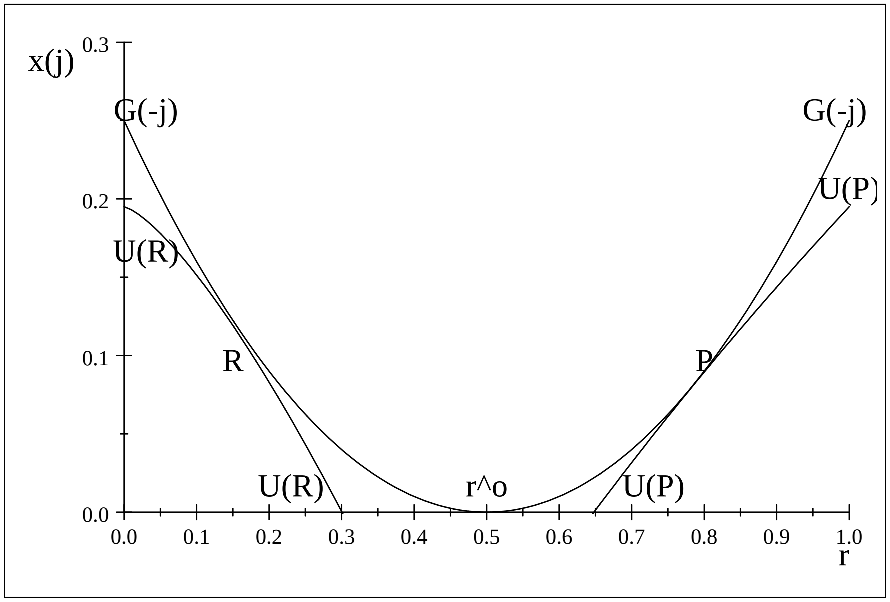


Figure 8: The Grossman-Helpman (2001) framework.

The equilibrium of this game is "a subgame perfect Nash equilibrium in the political competition between the groups, which means that the contribution schedule of each group must be an optimal response to the set of schedules of the others, when all groups correctly anticipate the policymaker's best response." (For a formal definition, used below, see Grossman and Helpman 2001: 250-1). Note that any competition between groups with opposing interests places the competing groups in a suboptimal situation. The government does not care where the contributions come from, so it is willing to change its policy in the direction of the higher bidder, whichever it is. Because they must thwart the influence of the opponents(s), the groups must spend resources even if in the end the policy does not move much or at all. Indeed, when at the margin the opposing groups spend equal amounts, the game is a prisoners' dilemma: the government extracts the contributions and the policy does not change at all from its peak preference (Dixit, Grossman, and Helpman 1997).

To solve for the equilibria, we need to specialize somewhat the government utility function. Assume that

$$G(r, r^o, \mathbf{x}(r)) = -(r - r^o)^2 + \nu \sum_j x^j(r), \tag{4}$$

where υ indicates the willingness of the government to move policy in exchange for contributions. Then, in any equilibrium, it must be true that

$$\hat{r} - r^o = (\nu/2)\sum_j \frac{\partial \hat{x}^j(r)}{\partial r} \tag{5}$$

Each group, in turn, offers a schedule of payments $\hat{x}^j(r)$ that optimally substitutes the marginal gains from increasing (for P) or decreasing (for R) rate of redistribution with the marginal cost of contributions according to

$$\frac{\partial \hat{x}^j(r)}{\partial r} = -\frac{\partial U^j(Y^j(r,x^j))/\partial r}{\partial U^j(Y^j(r,x^j))/\partial x^j}. \tag{6}$$

The post-redistribution utility of j is its net income, $Y^j(r,x^j)$, so that

$$U^j(Y^j(r,x^j)) = (1-r(x^j,x^{\neg j}))y^j + r(x^j,x^{\neg j})(1-\lambda r(x^j,x^{\neg j}))\overline{y} - x^j, \tag{7}$$

$$\frac{\partial U^j(Y^j(r,x^j))}{\partial r} = (1-2\lambda r)\overline{y} - y^j, \tag{8}$$

and

$$\frac{\partial U^j(Y^j(r,x^j))}{\partial x^j} = -1, \tag{9}$$

yielding[13]

$$\frac{\partial \hat{x}^j(\hat{r})}{\partial \hat{r}} = (1-2\lambda r)\overline{y} - y^j \tag{10}$$

Hence, in equilibrium

$$\hat{r}(\hat{x}^P,\hat{x}^R) = r^o + v((1-2\lambda\hat{r}(\hat{x}^P,\hat{x}^R))\overline{y} - \frac{1}{2}(y^P + y^R)). \tag{11}$$

Given that $r^o, \overline{y}, y^P$, and y^R are all functions of income inequality, measured by α or G, the equilibrium rate of redistribution can be written as

$$\hat{r}(G; v, \lambda) = \frac{r^o + v(\overline{y} - \frac{1}{2}(y^P + y^R))}{1 + v2\lambda\overline{y}}. \tag{12}$$

Now, every observed distribution of income is skewed to the right, which is sufficient to guarantee that for any $\overline{y}$, $y^R - \overline{y} > \overline{y} - y^P$ and

[13] Note that the marginal rate of substitution does not depend on the form of $U(.)$.

$\overline{y} - \frac{1}{2}(y^P + y^R) < 0$.[14] Hence, as long as $v > 0$, $\hat{r}(G; v, \lambda) < r^o$. The intuition behind this result is that the average rich agent has more to gain from decreasing r than the average poor from increasing r and is willing, therefore, to offer more at the margin: $-\frac{\partial \hat{x}^R(\hat{r})}{\partial \hat{r}} > \frac{\partial \hat{x}^P(\hat{r})}{\partial \hat{r}}$. Hence, $\sum_j \frac{\partial \hat{x}^j(r)}{\partial r} < 0$ and $\hat{r}(x^P, x^R) < r^o$. The effect of v is obvious: when the government is more willing to move the policy in response to contributions, the advantage of the rich increases and $\hat{r}(v)$ declines.

Figure 9 shows the rates of redistribution resulting from the competition for political influence when everyone votes, so that $r^o = r^M$, and when electoral participation follows $1 - 0.85G$, as in the section above, and r^o is given by Remark 1.

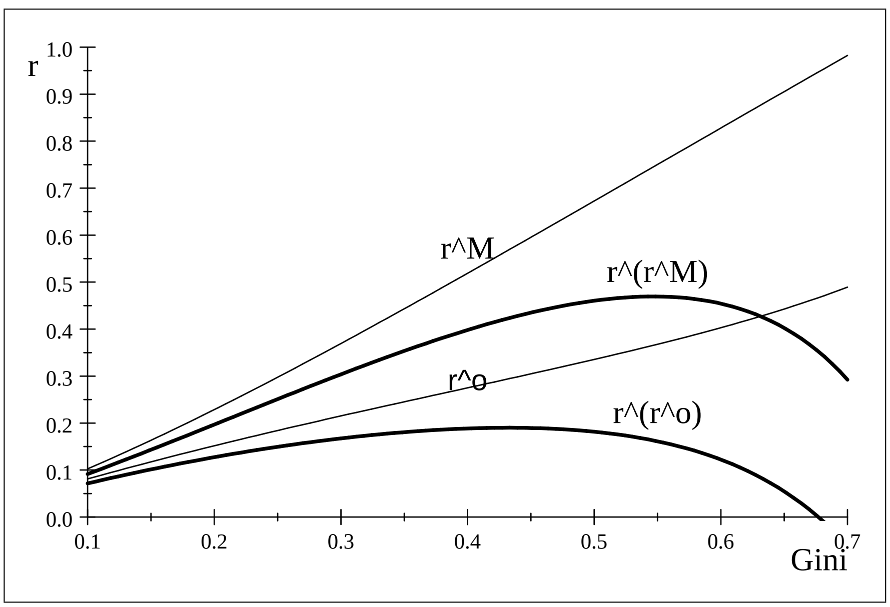


Figure 9: Equilibrium (thick lines) and median voter (thin lines) rates of redistribution by Gini coefficient of market income. r^M is the median voter's rate when everyone votes and the rate is determined only by elections; $\hat{r}(r^M)$ is the rate resulting from competition for influence when everyone votes; r^o is the rate preferred by the median among voters when electoral participation follows $1 - 0.85G$; $\hat{r}(r^o)$ is the rate resulting from competition for influence when the poor do not vote. $(v = 0.1)$

[14] Substituting for y^P and y^R, $\overline{y} - 0.5(y^P + y^R)$ can be rewritten as $\overline{y}(1 - 0.5\frac{1+y^o-2(y^o)^{1-\alpha}}{1-(y^o)^{-\alpha}})$, which is negative if $\frac{1+y^o-2(y^o)^{1-\alpha}}{1-(y^o)^{-\alpha}} > 2$ or if $y^o - 2\frac{y^o-1}{(y^o)^\alpha} > 1$. Now when $y^o = 1$, the $LHS = 1$. In turn $dLHS/dy^o = 1 + \frac{2}{(y^o)^{\alpha+1}}(y^o + \alpha - \alpha y^o) > 0$ for all $\alpha \geq 1$ because the sufficient condition is that $y^o + \alpha - \alpha y^o > 0$ or that $y^o < \frac{\alpha}{\alpha - 1}$, which is always true. Hence, $\overline{y} - 0.5(y^P + y^R) < 0$ for all α.

Hence, rates of redistribution are always lower than those that would prevail under perfect political equality and even lower than those preferred by the median among those who do vote. Moreover, when groups compete for political influence, rates decline when inequality becomes greater in already unequal societies. Effective political equality is not possible in economically unequal societies.

Returning to the relation between gross and net incomes shows that redistribution does mitigate somewhat the inequality of market incomes in less unequal societies but almost not at all in unequal ones. In particular, the relation between net and gross incomes when poor people do not vote and groups compete for political influence is almost exactly the same as the calibration of the influence function with $\eta = 0.8$, which almost perfectly fits the observed patterns.

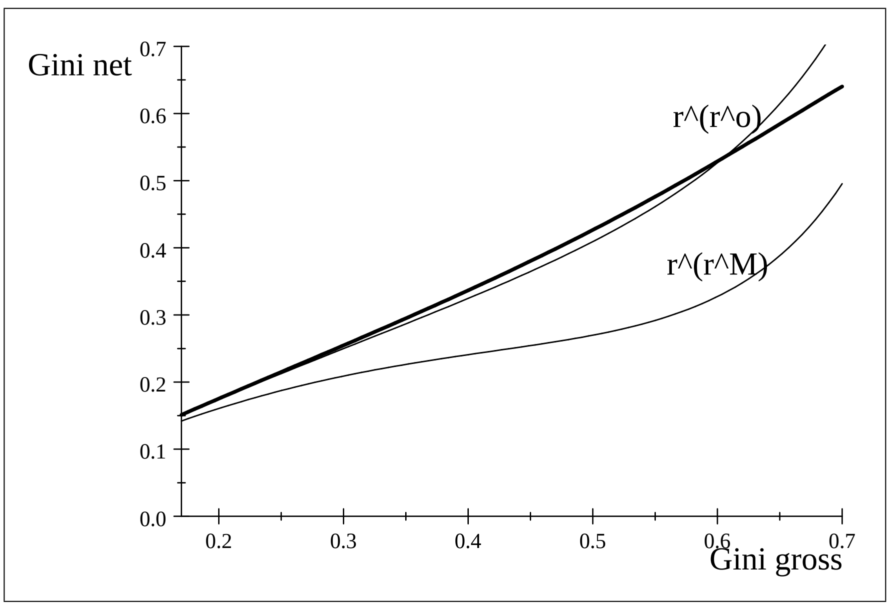


Figure 10: Gini of net incomes as a function of Gini of market incomes. $\hat{r}(r^M)$ assumes everyone votes, $\hat{r}(r^o)$ assumes that electoral turnout follows $1 - 0.85G$. The thick line is the prediction assuming the influence function is $w = y^{0.8}$.

This model of "class contra class" may be exaggerated. Some of the rich may believe that inequality generates negative externalities for them, some may have strong egalitarian beliefs, some may be just compassionate. The resistance of the rich against taxation seems to be higher in the United States than in several European countries, where the rich have learned to live with high rates. Incorporating this possibility into the model shows,[15] however, that to neutralize the inequality of resources this resistance would have to be minimal. And, at least in the United States, the very rich who support redistribution are few.[16]

Another important caveat is that governments may not be indifferent as to where the money comes from. Left-wing governments may value contributions from unions but not from large corporations or wealthy individuals, while right-wing parties may be happy to accept them. In one electoral campaign in Brazil, for example, when a newspaper reported that the Left-wing candidate Luis Ignacio "Lula" da Silva received a contribution from the largest construction company in the country, the cadres of the Workers' Party (PT) rose in indignation and forced him to return the money. Several Democratic candidates in the U.S. vaunted the fact that their funds were raised by small contributions. Suppose that $v = \{v^P, v^R\}$, where v^j is the government's value of contributions from group j, and that for a Left government $v^P > v^R$. Contributions from the rich are then perfectly neutralized by those of the poor, $\hat{r} = r^o = r^M$, when $\frac{v^P}{v^R} = \frac{y^R - y^M}{y^M - y^P}$. The value of this ratio is implausibly high in very unequal societies – 579 for $G = 0.7$ – but no longer so at lower leveles of inequality – 34 for $G = 0.5$ and 7.4 for $G = 0.2$. Hence, the impact of money on redistribution may be mitigated when Left parties are in government in societies that already have a relatively egalitarian distribution of market incomes, which seems to be true of the Scandinavian countries (Prat 1999).

5 Conclusion

An article in *The Encyclopedia of Public Choice* asserts that "the median voter model can be regarded not only as a convenient method of discussing majoritarian politics and a fruitful engine of analysis, but also *a fundamental property of democracy.*" (Congleton 2003: page). If only it were so .... In the most comprehensive study to date of the impact of popular preferences on policy outcomes, Gilens (2012: 4) summarizes his findings as follows:

[15] One way to think that some of the rich may not oppose redistribution is to assume that the average rich suffers less from being taxed, so that

$$U^R(Y^R(\hat{r}), x^R) = (1 - \beta\hat{r})y^R + \hat{r}(1 - \lambda\hat{r})\overline{y} - x^R, \beta \in (0, 1) \tag{13}$$

Solving for the equilibrium rate shows that $\hat{r} = r^o$ only if β is extremely low, so the rich almost do not care about being taxed. For example, when $G = 0.5, \beta = 0.05$.

[16] According to *Daily Kos*, March 29, 2015, "Someone finally polled the 1% –And it's not pretty," the proportion of the 1% who agree with the statement "Our government should redistribute wealth by heavy taxes on the rich" is 17%, as contrasted with 52% for the general public.

> What I find is hard to reconcile with the notion of political equality in Dahl's formulation of democracy. The American government does respond to the public's preferences, but that responsiveness is strongly tilted toward the most affluent citizens. Indeed, under most circumstances, the preferences of the vast majority of Americans appear to have essentially no impact on which policies the government does or doesn't adopt.

Economic inequality has multiple ways of infiltrating itself into politics. Citizens with different economic resources have unequal influence over government policies, whether in elections or when the specific policies are chosen and implemented. Equality of formal political rights is not sufficient to support effective political equality.

Redistribution of income through taxes and transfers is the policy to which the median voter model is most naturally and most frequently applied. Yet before something can be re-distributed, it must be first distributed: logically, the distribution of market incomes comes first. And, as Stigler (1975) observed, all policies – from credentialing nurses, to issuing taxi medallions, to prohibitions of noxious products – affect the distribution of incomes. A woman with two years of vocational education has a different earning capacity when anyone can become a nurse and when becoming one requires this training. In turn, incomes of all those who use nursing services are different when entry into nursing is open than when it is regulated. While this is just a minor example, the same is true of more consequential policies: regulation of natural monopolies, tariffs, regulation of labor markets, laws regarding consumer protection, environmental regulations, ...; the list is endless. Even when the state does not enter directly into private transactions, the terms of these transactions depend on public policies. Consider an example, due to Stiglitz (1994), of buying car insurance against theft. Consumers pay premiums and, if theft transpires, receive benefits. But the price of insurance – the terms of this private transaction between individuals and insurance companies – depends on the probability that the insured event occurs and in turn this probability depends on the number of policemen that the government puts on the street. The state is present in all private transactions.

All these policies are vulnerable to political influence. Moreover, many of them concentrate benefits to small group while spreading costs broadly and few of them are subject to retrospective electoral sanctions, so rational ignorance generates an even greater inequality of political influence with regard to such policies than with regard to the redistribution through the fisc. Hence, there are good reasons to suspect that the influence of economic resources over government policies is ubiquitous.

Something is wrong when a plurality of citizens in a democracy answer the question about which institutions have most power in their country with "banks."[17] Access of money to politics is the scourge of democracy.

6 References

Acemoglu, Daron, and James Robinson. 2000. "Why did the West extend the franchise? Democracy, Inequality, and Growth in Historical Perspective." *Quarterly Journal of Economics 115:* 1167-1199.

Alesina, Alberto and George-Marios Angeletos. 2005. "Fairness and Redistribution." *American Economic Review 95*: 960-980.

Ansell, Christopher, and David Samuels. 2010. "Democracy and Redistribution, 1880-1930: Reassessing the Evidence." Paper presented at the Annual Meeting of the American Political Science Association, Washington, D.C.

Bassett, William F., John P. Burkett, and Louis Putterman. 1999. "Income distribution, government transfers, and the problem of unequal influence." *European Journal of Political Economy 15*: 207-228.

Becker, Gary S. 1983. "A Theory of Competition Among Pressure Groups for Political Influence," *The Quarterly Journal of Economics 98:* 371-400.

Bénabou, Roland. 2000. "Unequal Societies: Income Distribution and the Social Contract." *American Economic Review 90:* 96-129.

Bénabou, Roland, and Efe A. Ok. 2001. "Social Mobility and the Demand for Redistribution: the PUOM Hypothesis." *Quarterly Journal of Economics 116:* 447-87.

Bernhagen, Patrick and Thomas Bräuninger. 2005. "Structural Power and Public Policy: A Signaling Model of Business Lobbying in Democratic Capitalism." *Political Studies 53:* 43-64.

Bertola, Giuseppe. 1993. "Factor Shares and Savings in Endogenous Growth." *American Economic Review 83:* 1184-1198.

Black, Duncan. 1948. "On the Rationale of Group Decision-making." *Journal of Political Economy 56*: 23-34.

[17] See Centro de Estudios Sociologicos (CIS), Madrid, Barómetro Noviembre 2010, Estudio no. 2.853. The question was "De las siguientes instituciones o colectivos, cuáles cree Ud. que tienen más poder en España?" (Of the following institutions or bodies, which do you believe have more power in Spain?). Banks were mentioned as most powerful by 31.6 percent of respondents, the government by 26.4 percent, large firms by 15.1 percent.

Campante, Filipe R. 2011. "Redistribution in a Model of Voting and Campaign Contributions." *Journal of Public Economics 95:* 646-656

Congleton, Roger. 2003. "The Median Voter Model." In C. K. Rowley and F. Schneider (Eds.). *The Encyclopedia of Public Choice.* Kluwer Academic Press.

Corneo, Giacomo, and Hans Peter Gruner. 2000. "Social Limits to Redstribution." *American Economic Review 90:* 1491-1507.

Dahl, Robert A. and Charles E. Lindblom. 1953. *Politics, Economics, and Welfare.* New York: Harper&Brothers.

De Luca, Giacomo, Roland Hodler, Paul A. Raschky, and Michele Valsecchi. 2015. "Ethnic Favoritism: An Axiom of Politics? CESIFO Working Paper No. 5209.

Dixit, Avinash, Gene M.Grossman, and Elhanan Helpman. 1997. "Common Agency and Coordination: General Theory and Application to Government Policy Making." *Journal of Political Economy 105:* 752-769.

Downs, A. 1957. *An Economic Theory of Democracy.* New York: Harper Collins

Fong, Christina, 2001. "Social Preferences, Self-Interest, and the Demand for Redistribution". Journal of Public Economics, LXXXII: 225-246.

Gaskins, Ben, Matt Golder, and David A. Siegel. 2013. "Religious Participation and Economic Conservatism." *American Journal of Political Science 57:* 823–840.

Gilens, Martin. 2012. *Affluence and Influence: Economic Inequality and Political Power in America.* Princeton: Princeton University Press.

Grossman, Gene M. and Elhanan Helpman. 1994. "Protection for Sale." *American Economic Review 84:* 833-850.

Grossman, Gene M. and Elhanan Helpman. 2001. *Special Interest Politics.* Cambridge, MA: MIT Press.

Habermas, Jurgen. 1975. *Legitimation Crises.* Boston: Beacon.

Harms, Philipp, and Stefan Zink. 2003. "Limits to redistribution in a democracy: a survey." *European Journal of Political Economy 19:* 651-668.

Holmes, Stephen and Cass R. Sunstein. 1999. *The Cost of Rights.* New York: W.W. Norton.

Huber, John D. and Piero Stanig. 2007. "Why do the poor support right-wing parties? A cross-national analysis." Ms. Department of Political Science, Columbia University.

Kasara, Kimuli and Pavithra Suryanarayan. 2015. "When Do the Rich Vote Less Than the Poor and Why? Explaining Turnout Inequality across the World." *American Journal of Political Science 59:* 613-627.

Kelly, Nathan J. and Peter K. Enns. 2010. "Inequality and the Dynamics of Public Opinion: The Self-Reinforcing Link Between Economic Inequality and Mass Preferences." *American Journal of Political Science 54*: 855–870.

Kelsen, Hans. 1988 [1929]. *La Démocratie. Sa Nature-Sa Valeur.* Paris: Economica.

Krueger, Ann. 1974. "The Political Economy of Rent Seeking." *American Economic Review 64:* 291-303.

Lind, Jo T. 2005. "Why is There so Little Redistribution?" *Nordic Journal of Political Economy 31:* 111-125.

McCarty, Nolan, and Jonas Pontusson. 2008. "The Political Economy of Inequality and Redistribution.". In Wiemer Salverda, Brian Nolan and Timothy Smeeding (eds). *Handbook of Economic Inequality.* Oxford University Press. Pages 665-692.

Meltzer, Allan H., and Scott F. Richard. 1981. "A Rational Theory of the Size of Government." *Journal of Political Economy 89:* 914-27.

Mill, John Stuart. 1989 [1859]. *On Liberty and Other Writings.* Edited by Stefan Colini. Cambridge: Cambridge University Press.

Moene, Karl Ove and Michael Wallerstein. 2001. "Inequality, Social Insurance, and Redistribution." *American Political Science Review 95*: 859-74.

Niskanen, William. 1971. *Bureaucracy and Representative Government.* Chicago: Aldine-Atherton.

Peltzman, Sam. 1976. "Toward a More General Theory of Regulation." With "Comments" by Jack Hirschleifer and Gary Becker. *Journal of Law and Economics 19:* 211-248.

Piketty, Thomas. 1995. "Social Mobility and Redistributive Politics." *Quarterly Journal of Economics 110:* 551-584.

Posner, Richard A. 1987. "The Constitution as an Economic Document." *The George Washington Law Review 56:* 4-38.

Prat, Andrea. 1999. "An Economic Analysis of Campaign Financing." Working paper. Tilburg University.

Przeworski, Adam. 2014. "PIPE: Political Institutions and Political Events." https://sites.google.com/a/nyu.edu/adam-przeworski/

Przeworski, Adam, Gonzalo Rivero, and Tianyang Xi. 2015. "Elections as a Method of Processing Conflicts." *European Journal of Political Economy 39*: 235-248.

Putterman, Louis. 1996. "Why Have the Rubble not Redistributed the Wealth? On the Stability of Democracy and Unequal Wealth." In John E. Roemer (ed.), *Property Relations, Incentives, and Welfare.* London: McMillan. Pages 359-389.

Rae, Douglas W. 1969. "Decision Rules and Individual Values in

Constitutional Choice." *American Political Science Review 63:* 40-56.

Rae, Douglas W. 1971. "Political Democracy as a Property of Political Institutions" *American Political Science Review 65:* 111-119.

Roemer, John E. 1998. "Why the poor do not expropriate the rich? An old argument in new garb." *Journal of Public Economics 70:* 399-424.

Roberts, Kevis W.S. 1977. "Voting over Income Tax Schedules." *Journal of Public Economics 8*: 329-40

Romer, Thomas. 1975. "Individual Welfare, MajorityVoting, and the Properties of a Linear Income Tax." *Journal of Public Economics 14*: 163-85.

Rosendorf, B. Peter. 2001. "Choosing Democracy." *Economics and Politics 13*: 1-31.

Rothstein, Paul. 1991. "Representative voter theorems." *Public Choice 72:* 193-212.

Solt, Frederick. 2009. "Standardizing the World Income Inequality Database." *Social Science Quarterly 90(2)*: 231-242.

SWIID. 2011. "The Standardized World Income Inequality Data Base." Version 3.1.

Stegmueller, Daniel. 2013. "Religion and Redistributive Voting in Western Europe." *Journal of Politics 75:* 1064–1076.

Salt, Frederick. 2006. "Economic Inequality and Democratic Political Engagement." Department of Political Science, Southern Illinois University, Carbondale.

Sen, Amartya. 1988. "Freedom of Choice: Concept and Content." *European Economic Review 32:* 269-94.

Stigler, George. 1971. "The Theory of Economic Regulation." *Bell Journal of Economics and Management Science 3:* 3-18.

Stigler, George. 1975. *The Citizen and the State.* Chicago: University of Chicago Press.

Stiglitz, Joseph E. 1994. *Whither Socialism?* Cambridge, MA: MIT Press.

Thistle, Paul D. 1989. "Ranking Distributions with Generalized Lorenz Curves." *Southern Economic Journal 56*: 1-12.

7 Appendix: Proofs of Proposition 2

7.1 Lognormal.

Consider a log-normal distribution of income, with median 1 and mean $\exp(\alpha^2/2)$, $\log y \sim \mathcal{N}(0, \alpha)$, and a political influence function $w = y^\eta, \eta \geq 0$. Let F_α be the c.d.f. for the distribution of y. The decisive agent has income $y^D(\eta,\alpha)$ defined by

$$\int_0^{y^D(\eta,\alpha)} y^\eta dF_\alpha(y) = \int_{y^D(\eta,\alpha)}^{\infty} y^\eta dF_\alpha(y).$$

Let $\hat{y}(\alpha) := y^D(1,\alpha)$. Note that $\log y^\eta \sim \mathcal{N}(0,\eta\alpha)$,which implies that y^η has a c.d.f. $F_{\eta\alpha}$. Therefore,

$$y^D(\eta,\alpha) = \hat{y}(\eta\alpha).$$

Given equation (3), the ideal rate of redistribution of an agent with pre-fisc income y^i is

$$r^i(y^i,\alpha) := \min\left\{\max\left\{\frac{1}{2\lambda}\left(1-\frac{y^i}{\exp\left(\frac{\alpha^2}{2}\right)}\right),0\right\},1\right\}.$$

Therefore, the equilibrium rate of redistribution is

$$r^D(\eta,\alpha) := \hat{r}(\hat{y}(\eta\alpha),\alpha).$$

Study now how $r^D(\eta,\alpha)$ responds to changes in α, using the fact that for any $\alpha \in \mathbb{R}_{++}$, $\hat{y}(\alpha) = \exp(\alpha^2)$. Hence, $\hat{y}(\eta\alpha) = \exp(\eta^2\alpha^2)$, and

$$r^D(\eta,\alpha) = \min\left\{\max\left\{\frac{1}{2\lambda}\left(1-\exp\left(\left(\eta^2-\frac{1}{2}\right)\alpha^2\right)\right),0\right\},1\right\}.$$

Therefore, $r^D(\eta,\alpha)$ is strictly increasing with α if and only if

$$\eta < \frac{\sqrt{2}}{2} \approx 0.71.$$

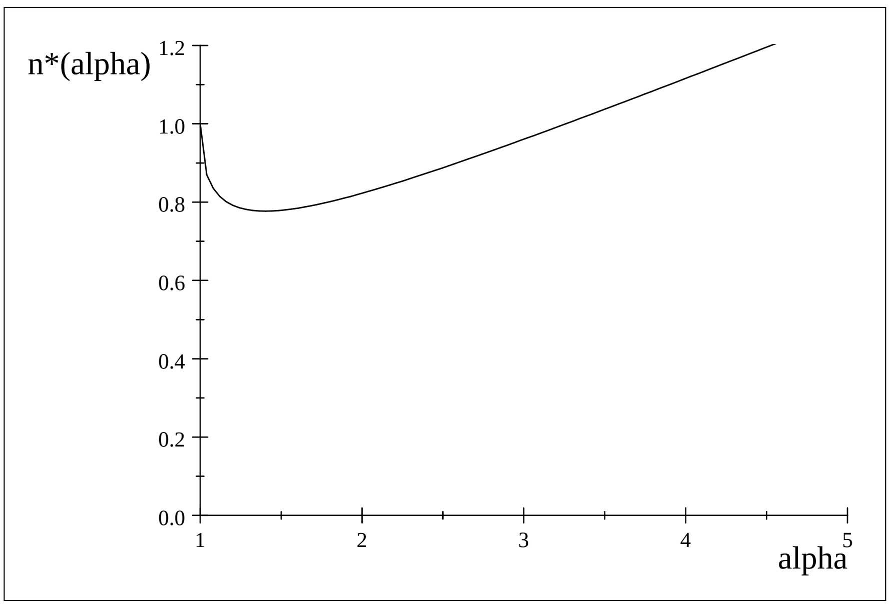
7.2 Pareto

Consider a Pareto distribution of income, $F(y) = p(y) = 1 - y^{-\alpha}, y \geq 1, \alpha > 1$ and a political influence function $w = y^\eta, w(1) = 1$, $\alpha > \eta \geq 0$. Note that income distribution is more equal when α is larger. The mean is $\alpha/(\alpha-1)$, the median income $2^{1/\alpha}$, so that $y^M/\overline{y} = (2^{1/\alpha})/(\frac{\alpha}{\alpha-1})$. The Gini coefficient of a Pareto distribution is $G(\alpha) = (2\alpha-1)^{-1}$.

The proportion of political influence of agents at or below p is $L(p) = 1-(1-p)^{(\alpha-\eta)/\alpha}$. Given that the decisive agent is the one for whom $L(p^D) = 0.5$, $p^D = 1 - 0.5^{\alpha/(\alpha-\eta)}$.

Under perfect political equality, $\eta = 0$, the decisive agent is the one with median income. Given an income distribution, as political inequality becomes more sensitive to economic inequality, the decisive agent is located in a higher percentile of income distribution. This is because $E(w(y)) = \int_1^w w(y) f_Y(y)dy = \int_1^w y^n \frac{\alpha}{y^{\alpha+1}}dy = \alpha \int_1^w y^{\eta-\alpha-1}dy$. Hence, $L(y) = \int_1^w \alpha y^{\eta-\alpha-1}dy / \int_1^\infty \alpha y^{\eta-\alpha-1}dy = 1 - y^{\eta-\alpha}$. Given that $y^{\eta-\alpha-1} = 1 - p(y)$, $L(p) = 1 - (1-p)^{(\alpha-\eta)/\alpha}$. $L(p^D) = 0.5$ implies $p^D = 1 - 0.5^{\alpha/(\alpha-\eta)}$. When $\eta = 0$, $p^D = 0.5$. In turn, $\frac{\partial p^D}{\partial \eta} = -(1 - p^D)\frac{\alpha}{(\alpha-\eta)^2}\ln 0.5 > 0$.

The derivative $\frac{\partial r^D}{\partial \alpha} = \frac{\partial}{\partial \alpha}\frac{1-\frac{\alpha-1}{\alpha}0.5^{1/(\eta-\alpha)}}{2\lambda} = \frac{1}{2\lambda}\frac{\alpha(\alpha-1)(-\ln 0.5)-(\alpha-\eta)^2}{0.5^{\frac{1}{\alpha-\eta}}\alpha^2(\alpha-\eta)^2}$.The numerator $\alpha(\alpha-1)(-\ln 0.5)-(\alpha-\eta)^2 \lesseqqgtr 0$ when $\eta \lesseqqgtr \alpha - \sqrt{-\ln 0.5}\sqrt{\alpha(\alpha-1)}$. Let $\eta^*(\alpha)$ denote the value of $\eta(\alpha)$ for which $\frac{\partial r^D}{\partial \alpha} = 0$. The function $\eta^*(\alpha) = \alpha - \sqrt{-\ln 0.5}\sqrt{\alpha(\alpha-1)}$ is convex to the origin, with $\eta(1) = 1$ and a minimum of 0.78 when $\alpha = 1.4$



Hence, if $\eta < \min_\alpha \eta^*(\alpha)$, $\frac{\partial r^D}{\partial \alpha} < 0$, $\forall \alpha$, that is, the rate of redistribution declines in equality for all distributions of income. If $\min_\alpha \eta^*(\alpha) < \eta < 1$,this rate decreases in inequality except for very equal and very unequal distributions of income. If $\eta > 1$, the rate increases in inequality in very equal societies and then decreases.